PHY4410 HW 5 Fall 2019. Due Monday Nov 4 by 4:30pm

1 . Show that the average energy, defined as

$$U = < \varepsilon_s > = \frac{\sum_S \varepsilon_s \exp\,(-\varepsilon_s/\tau)}{Z},$$

where $Z = \sum_S \exp\,(-\varepsilon_s/\tau)$,

can also be expressed as

$$U = \tau^2 \frac{\partial \ln(Z)}{\partial \tau}$$

.

2. Consider a system with two energy states, one with energy 0 and the other with energy ε. The partition function for the system is given by

$$Z = \exp\,(-0/\tau) + \exp\,(-\varepsilon/\tau) = 1 + \exp\,(-\varepsilon/\tau).$$

a) Find the expression for the average energy $U = < \varepsilon_s >$.
b) Using Desmos or some other program, plot the function U/ε on the y-axis and τ/ε on the horizontal axis.

3. We define the **heat capacity** of a system at constant volume to be

$$C_V \equiv \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_V$$

where σ is the statistical entropy $\sigma = \ln(g)$.
a) Show that this expression is equivalent to the alternate form

$$C_V = \left(\frac{\partial U}{\partial \tau}\right)_V$$

(Hint: use the "thermodynamic identiy" we will discuss in class)
b) For the average energy U found in problem 2a, find the expression for the corresponding heat capacity.
c) As in 2b above, plot your expression for C_V vs. τ/ε.

4. "Particle" in a box: A single atom of mass *m* in a cubical box with sides of length *L* has a wave function which forms a set of standing waves in the *x, y,* and *z* directions, such that the energy of a state with quantum numbers $n_x, n_y,$ and n_z is given by

$$\varepsilon_{\vec{n}} = \frac{1}{2m}\left(\frac{\hbar\pi}{L}\right)^2 \left(n_x^2 + n_y^2 + n_z^2\right)$$

The partition function *Z* is the sum over all possible states

$$Z = \sum_{n_x}\sum_{n_y}\sum_{n_z} \exp\left[-\hbar^2\pi^2\left(n_x^2 + n_y^2 + n_z^2\right)/2mL^2\tau\right]$$

a) Replace these sums with integrals from 0 to infinity, and evaluate the expression to find the partition function. (Note that you should find $Z \propto \tau^{3/2}$.)
b) Using your expression from a, show that the average energy U is given by

$$U = \frac{3}{2}\tau$$

(Hint: Use the expression for U from Problem 1.) This is the well-known result for the energy per atom of an ideal gas.