## PHY2250, Dr. Hawley: AC Operation of a Common-Emitter Amplifier

We've previously discussed the DC operation of a common-emitter amplifier, DC load lines, Q points, etc. We now consider the AC operation of such an amplifier. We're going to put two resistors on the emitter, along with a bypass capacitor as shown:

( $\mathrm{R}_{\mathrm{EI}}$ can either be an explicit resistor or it can represent the effective "resistance" which appears in real-world performance of diodes, i.e. the fact that there is a finite slope to the line on current-vs.-voltage graph for a diode, rather than an "instant" transition at 0.7 V .)

Let's break all quantities into their DC and AC parts, which will be denoted by a subscript "Q" and a lower-case letter, respectively. So, e.g., $\mathrm{V}_{\mathrm{E}}=\mathrm{V}_{\mathrm{EQ}}+\mathrm{V}_{\mathrm{E}}$, as shown in the following graph:


Note that because of the "diode drop" between the base and the emitter, $\mathrm{V}_{\mathrm{E}}=\mathrm{V}_{\mathrm{B}}+0.7 \mathrm{~V}$. The "AC part" of this equation tells us that $\mathrm{v}_{\mathrm{E}}=\mathrm{v}_{\mathrm{B}}$. A simple proof follows:

$$
\begin{align*}
\mathrm{V}_{\mathrm{E}} & =\mathrm{V}_{\mathrm{B}}-0.7 \mathrm{~V} \\
\mathrm{~V}_{\mathrm{EQ}}+\mathrm{v}_{\mathrm{E}} & =\mathrm{V}_{\mathrm{BQ}}+\mathrm{v}_{\mathrm{B}}-0.7 \mathrm{~V} \\
\mathrm{~V}_{\mathrm{EQ}}+\mathrm{v}_{\mathrm{E}} & =\left(\mathrm{V}_{\mathrm{EQ}}+0.7 \mathrm{~V}\right)+\mathrm{v}_{\mathrm{B}}-0.7 \mathrm{~V} \\
\mathrm{v}_{\mathrm{E}} & =\mathrm{v}_{\mathrm{B}} . \tag{1}
\end{align*}
$$

Because of bypass capacitor $\mathrm{C}_{\mathrm{E}}$ allows AC signals to go "around" $\mathrm{R}_{\mathrm{E} 2}$, the AC part of the emitter voltage, $\mathrm{v}_{\mathrm{E}}$, is given by

$$
\begin{equation*}
v_{E}=i_{E} R_{E 1} . \tag{2}
\end{equation*}
$$

The collector voltage, $\mathrm{V}_{\mathrm{C}}$, is given by

$$
\mathrm{V}_{\mathrm{C}}=\mathrm{V}_{\mathrm{CC}}-\mathrm{I}_{\mathrm{C}} \mathrm{R}_{\mathrm{C}}
$$

$\mathrm{V}_{\mathrm{CQ}}+\mathrm{v}_{\mathrm{C}}=\mathrm{V}_{\mathrm{CC}}-\left(\mathrm{I}_{\mathrm{CQ}}+\mathrm{i}_{\mathrm{C}}\right) \mathrm{R}_{\mathrm{C}}$,
and the "AC part" of this means that

$$
\begin{equation*}
\mathrm{v}_{\mathrm{C}}=-\mathrm{i}_{\mathrm{C}} \mathrm{R}_{\mathrm{C}} . \tag{3}
\end{equation*}
$$

We are now ready to compute the voltage gain of this amplifier.

The AC Voltage Gain $\mathbf{A}_{\mathbf{V}}$ is defined to be the ratio of the AC parts of output voltage to input voltage,

$$
\mathrm{A}_{\mathrm{V}}=\mathrm{v}_{\text {out }} / \mathrm{v}_{\text {in }} .
$$

Considering that $v_{\text {in }}=v_{B}=v_{E}$, and $v_{\text {out }}=v_{C}$, we find

$$
A_{V}=v_{C} / v_{E}=-i_{C} R_{C} / i_{E} R_{E I} .
$$

Given that the AC beta $\beta \equiv \mathrm{i}_{\mathrm{C}} / \mathrm{i}_{\mathrm{B}}$ is a large number (close to the value of $\beta_{D C} \equiv \mathrm{I}_{\mathrm{C}} / \mathrm{I}_{\mathrm{B}}$, which is on the order of 100 ), $\mathrm{i}_{\mathrm{C}}$ and $\mathrm{i}_{\mathrm{E}}$ are approximately equal. Therefore the gain of the amplifier is

$$
A_{V}=-\frac{R_{C}}{R_{E 1}} .
$$

So by lowering the value of $\mathrm{R}_{\mathrm{EI}}$, we can increase the gain of the amplifier.
What then is the function of the resistor $\mathrm{R}_{\mathrm{E} 2}$ ? It affects the DC operation of the amplifier, i.e. it controls the placement of the Q point on the DC load line. Recall that $I_{E Q}=V_{E Q} /\left(R_{E 1}+R_{E 2}\right)$, and since $I_{E}$ is approximately equal to $I_{C}$,

$$
\begin{aligned}
\mathrm{V}_{\mathrm{C}} & =\mathrm{V}_{\mathrm{CC}}-\mathrm{I}_{\mathrm{C}} \mathrm{R}_{\mathrm{C}} \\
& =\mathrm{V}_{\mathrm{CC}}-\mathrm{V}_{\mathrm{EQ}} \mathrm{R}_{\mathrm{C}} /\left(\mathrm{R}_{\mathrm{E} 1}+\mathrm{R}_{\mathrm{E} 2}\right) .
\end{aligned}
$$

So the combination $R_{E 1}+R_{E 2}$ takes the place of the single $R_{E}$ in our previous studies of the DC characteristics of the amplifier.

