## PHY2010 HW 3 Fall 2013-Answers

Use $345 \mathrm{~m} / \mathrm{s}$ for the speed of sound

1. In a given string, the tension is 200 N and the mass per unit length is $0.1 \mathrm{~kg} / \mathrm{m}$. What is the wave speed?
a) $20 \mathrm{~m} / \mathrm{s}$
b) $45 \mathrm{~m} / \mathrm{s}$
c) $450 \mathrm{~m} / \mathrm{s}$
d) $2000 \mathrm{~m} / \mathrm{s}$

$$
v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{200 \mathrm{~N}}{0.1 \mathrm{~kg} / \mathrm{m}}}=44.7 \mathrm{~m} / \mathrm{s} \approx 45 \mathrm{~m} / \mathrm{s}
$$

2. A string on a particular bass instrument as a length of 80.0 cm , a mass per unit length of $0.50 \mathrm{~g} / \mathrm{m}$, and its fundamental frequency is 200 Hz . What is the tension in the string?
a) 51.2 N
b) 63.4 N
c) 78.5 N
d) 98.2 N

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\begin{aligned}
& f_{N}=\frac{N}{2 L} \sqrt{\frac{T}{\mu}}, N=1 \\
& \mu(2 L f)^{2}=T=(0.0005 \mathrm{~kg} / \mathrm{m})(2(0.8 \mathrm{~m})(200 \mathrm{~Hz}))^{2}=51.2 \mathrm{~N}
\end{aligned}
$$

3. A pipe 50 cm long has one end open and one end closed. If the air is at room temperature, what are the frequencies of the first 3 harmonics?

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\begin{aligned}
& f_{N}=\frac{N v}{4 L}, N=1,3,5, \ldots \\
& f_{1}=\frac{(1) 345 \mathrm{~m} / \mathrm{s}}{4(0.5 \mathrm{~m})}=172.5 \mathrm{~Hz} \\
& f_{3}=3 f_{1}=517.5 \mathrm{~Hz} \\
& f_{5}=5 f_{1}=862.5 \mathrm{~Hz}
\end{aligned}
$$

4. We can model a flute as a tube with both ends open, and a clarinet as a tube with one end open and one closed. If they have the same length...
a. What is the ratio of the fundamental frequency of the flute to that of the clarinet?
a) $2: 1$
b) $1: 2$
c) $3: 1$
d) $4: 1$
b. What is the difference in the harmonic series generated by each instrument?

The clarinet has only odd harmonics, whereas the flute has both even and odd harmonics.
5. A 2 m long "Kundt's tube" shows dust gathered in three places:


If the speed of sound in air is $345 \mathrm{~m} / \mathrm{s}$, what is the period of the sound this tube was playing?
a) 1.56 ms
b) 2.37 ms
c) 2.84 ms
d) 3.86 ms

You can fit 1.5 wavelengths into this pipe and have the nodes match the accumulations of dust. so $L=1.5 \lambda$. Since $v=f \lambda$ and $f=1 / T$, the period $T$ is given by

$$
T=\frac{\lambda}{v}=\frac{L / 1.5}{345 \mathrm{~m} / \mathrm{s}}=\frac{2 / 1.5}{345}=3.86 \mathrm{~ms} .
$$

6. Michael Hedges (cf. www.nomadland.com) tuned his guitar strings way, way down on occasion. If he decreased the tension on the heaviest string by a factor of 8 , what is the ratio of the new frequency to the old frequency?
Since one of Mersenne's Laws tells us $f$ is proportional to the square root of the tension, decreasing the tension by a factor of 8 will decrease the frequency by a factor of $\operatorname{sqrt}(8)$. So the ratio of the new to the old will be $\operatorname{sqrt}(1 / 8)=0.35$, which is choice $b$ ).
7. When a whip is cracked, the pulse accelerates until it is traveling at supersonic speed. Describe the physics that make this possible.


As the whip tapers, the mass per unit length $\mu$ gets smaller and smaller. Since the wave speed is inversely proportional to the square root of $\mu$, the wave speed increases without bound.
8. Ben is playing at The Anchor, but unfortunately nobody can hear his intricate bass playing because the low end is muddied by the resonant frequencies of the room/barn. Looking around, you estimate the length of the room to be 5 m . Assuming the air is at room temperature, and regarding the room as a pipe with both ends closed, what is the corresponding frequency of the first overtone?
a) 6.9 Hz
b) 17.25 Hz
c) 34.5 Hz
d) 69.0 Hz
9. (from Michael Fowler) Humans can only hear sounds in the frequency range 20 Hz to $20,000 \mathrm{~Hz}$. What would be the longest and shortest organ pipes there is any point in manufacturing? Assume the pipes are open at both ends.

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\begin{aligned}
& \mathrm{f}=\mathrm{Nv} / 2 \mathrm{~L} \rightarrow \mathrm{~L}=\mathrm{Nv} /(2 \mathrm{f}) \\
& \mathrm{L}_{\text {small }}=(1)(345) /[2(20,000)]=0.008625 \mathrm{~m}=8.6 \mathrm{~mm} \\
& \mathrm{~L}_{\text {big }}=(1)(345) /[2(20)]=8.6 \mathrm{~m}
\end{aligned}
$$

10. Berg \& Stork, Chapter 3, Problem 5: A certain stretched string has a fundamental frequency of 175 Hz . What is the new frequency if one
a. Decreases the tension by a factor of 4

Since frequency is proportional to $\operatorname{sqrt}(T)$, the frequency will decrease by a factor of 2 . So $f=87.5 \mathrm{~Hz}$
b. Doubles the length

Since frequency is proportional to $1 / L$, the frequency will decrease by a factor of 2 . So $f=87.5 \mathrm{~Hz}$
c. Decreases the mass per unit length by a factor of 9

Since frequency is proportional to $\operatorname{sqrt}(\mu)$, frequency will increase by a factor of 3 .
So $f=525 \mathrm{~Hz}$
d. Triples the diameter.

Mass per unit length goes as the density of the material times the cross-sectional area, which is proportional to the square of the diameter. Thus the mass per unit length will increase by a factor of 9 . And given our answer to part $c$, this means the frequency will decrease by a factor of 3 . So $f=58.3 \mathrm{~Hz}$

