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Fourier Transforms, Audio Engineering, and the Quantum Nature of Reality

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ABSTRACT

A short interdisciplinary, educational survey is presented, illustrating ways in which audio spectral analysis and quantum physics are intimately related. A basic conceptual understanding of Fourier transforms and their applications in audio engineering is sufficient to grasp aspects of quantum wave packets, the Heisenberg Uncertainty Principle, and more. Similarly, concepts from quantum mechanics can inform the understanding of audio effects such as aliasing, convolutions and wavelet transforms. The presenter is a computational physicist who authored a computer audio analysis suite for audio engineering students, noting several interdisciplinary connections in the process.

1. OVERVIEW OF FOURIER TECHNIQUES

1.1. Fourier Essentials

In audio engineering, one typically sees a signal referred to by its time domain representation *v(t)* (e.g., a waveform display in a Digital Audio Workstation of microphone voltage vs. time) and/or its frequency representation or ``frequency spectrum" *A*(*f*) (e.g., the display of a real-time analyzer). One representation can be exchanged for the other via the mathematical operations developed by Jean Baptiste Joseph Fourier for the study of heat flow [1]. Although a *Fourier transform* of the real-valued signal *v*(*t*) actually results in a complex-valued function $\underline{V}(f)$, many applications in audio engineering display only the magnitude of the latter, i.e. $A(f)=|\underline{V}(f)|$, while the phase information is not displayed. One justification for failure to display phase information can be traced to *Ohm's Law of Hearing* [2], the claim that the human auditory system is insensitive to variations of phase in the harmonics of complex tones.

Figure 1 shows some examples of signals and their Fourier transforms. We see in row (a) a sine wave, which comprises one frequency and has nontrivial values on the domain of all *t*, i.e. has *support* of all *t.* Row (b) shows in some sense the opposite extreme, namely of a "spike" or "impulse" signal, whose mathematical ideal is a Dirac delta function. We see the familiar result that this signal in some sense "contains all frequencies" as seen in its Fourier transform graph; thus the utility of an "impulse response", e.g. in room reverberation studies, for encapsulating the entire frequency spectrum. Finally row (c) shows a "Gabor wavelet," which is a cosine multiplied by a gaussian.

This is a moderately time-localized and bandwidth-limited signal that serves as a useful depiction of a quantum mechanical wave-particle "wave packet."

In general, we make the observation that *the more 'localized' a signal is in time, the more 'spread out' it is in frequency, and vice versa.* Mathematically, we can express the signal's support in time as Δt and in frequency as Δf, and write

$$\Delta f \, \Delta t \geq \varepsilon, \tag{1.1}$$

for some constant ϵ. As we will see, this is the central observation that underlies the Heisenberg Uncertainty Principle in quantum physics.

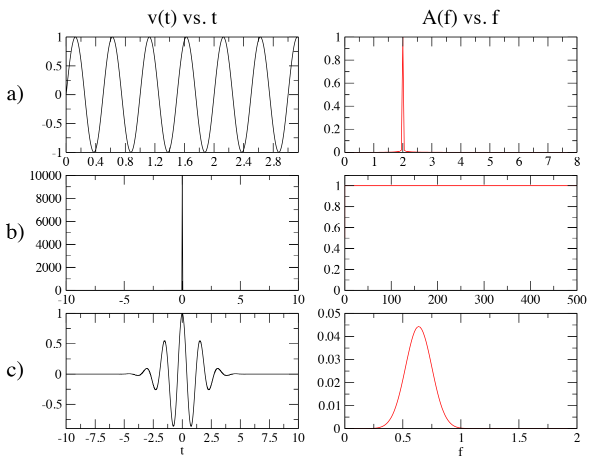

Figure 1. *Examples of Fourier transforms*. The signals in the left column are real-valued functions *v(t)*, and those in the right show the *magnitude* of the Fourier transform $A(f) = |\underline{V}^*\underline{V}|$. a) A sine wave transforms to a Dirac delta function, i.e. a sine wave has only one frequency. b) A delta function transforms to a flat line, that is to say, "an impulse contains all frequencies." c) A Gabor (or Morlet) wavelet, which is the product of a cosine wave and a gaussian, transforms to a gaussian.

1.2. Short Time Fourier Transforms, and Wavelets

One of the most common displays for audio engineering is the so-called short time Fourier transform (STFT), in which a running calculation of the Fourier spectrum is made using a short "frame" of data preceding (and, when possible, following) each time in the signal. This can then be used to produce a *spectrogram* image of the sound.

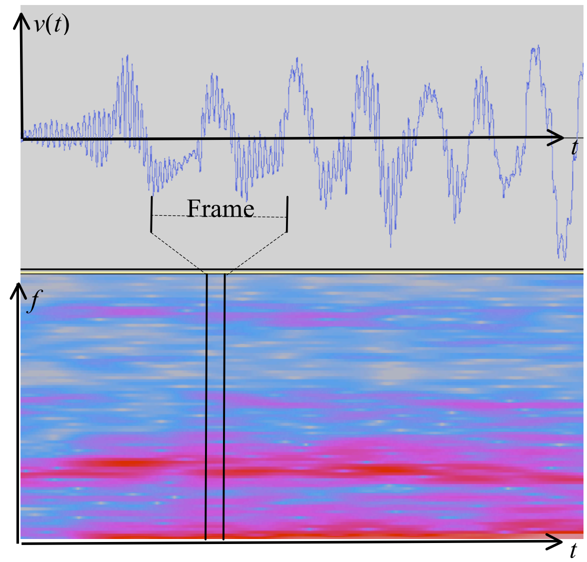

Figure 2. *Sample Spectrogram*. Time series data in the top panel are broken into a series of "frames" of finite length; the Fourier transform of the data in each frame is plotted as a vertical slice in the lower panel, in which amplitude is shown via color and frequency runs upward from zero. The weighting or *window* function applied over each frame can have mathematical similarities to forms which arise in quantum mechanics.

Typically the data in each frame is weighted evenly, however a weighting or *window* function may be used to give greater weight to data values near the center time for each frame. One windowing function with several desirable properties is a Gaussian shape, resulting in what is known as a *Gabor transform*. It was Dennis Gabor, a physicist, who had made this application to STFTs based on the idea of a "wave packet" arising in quantum mechanics [3] which we discuss below. Specifically, taking the sinusoidal shape used in the Fourier transform, and multiplying by a Gaussian results in a Gabor wavelet (also known as a Morlet wavelet based on Jean Morlet's application of Gabor's wavelet to the field of seismology), shown in Figure 1c. Several varieties of wavelets exist, but they all share the property that they are localized (e.g., in time) and are thus of great utility in reducing and representing transient phenomena such as seismological events and medical electrocardiogram (EKG) measurements. Furthermore, there exist digital "fast wavelet transform" methods for discrete datasets, which are *O(N)* in computational cost, making them substantially faster than the *O(NlogN)* FFT algorithm, however their applications to musical audio signal processing are not nearly as widespread as Fourier techniques [4].

2. QUANTUM PHYSICS

2.1. History of Quantum Mechanics

The history of the development of quantum mechanics can be placed in close analogy with the study of electromagnetism [4]. The central question from the times of Newton and Huygens concerned the nature of light: "is it a wave, or a particle?" A general consensus emerged after many years that the answer is "both." Light behaves like a classical wave under some circumstances, and like a classical particle under others. This culminated in Einstein's Nobel-prize-winning explanation of the Photoelectric Effect, which relied on Planck's quantization of light as bundles (later called "photons") with energy $E = hf$ and momentum $p = h/\lambda$, where h is Planck's constant of 6.63×10^{-34} m^2kg/s, and where f and λ are the frequency and wavelength of the (wave-like) light, respectively.

In 1924, Louis de Broglie hypothesized that matter should have a similar particle-wave duality, and by analogy to light he used the classical particle momentum p to define a wavelength λ for these "matter waves," known as the de Broglie Wavelength:

$$\lambda = \frac{h}{p}. \tag{2.1}$$

Three years later, de Broglie's hypothesis was confirmed in the electron diffraction experiments of Davisson and Germer at Bell Labs and independently by Thompson in Scotland. Note that h is a very small number, so for typical matter the wavelength is extremely short; the electrons in the aforementioned experiments had wavelengths on the order of those of x-rays. Heavier matter would tend to have even shorter wavelengths, indicating the rarity with which wave-like behavior is observed for macroscopic objects.

In the meantime, the mechanics describing the evolution of de Broglie's matter waves were given a mathematical basis by Erwin Schrödinger, who followed the analogy of electromagnetism [5] to arrive a complex-valued version of the heat equation (cue the tie-in with Fourier). Like the theory of electromagnetic and acoustical waves, the *Schrödinger equation* is a *linear* wave equation, in which superposition holds and thus Fourier techniques are useful.

2.2. The Wavefunction and Wave Packets

The Schrödinger equation describes the evolution of a complex-valued "wavefunction" $\psi(x,t)$, although often we may be interested in solutions which are "separable" in time & space, and thus we may often work in terms of the time-independent function $\psi(x)$. Direct interpretation of the wavefunction as a physical quantity is somewhat challenging; one typically remarks that the wavefunction provides a sort of probability density for the emergence of particle-like properties. Physical interpretations are made via *observables*, which serve as operators to 'extract' information encoded in the wavefunction. In analogy with object-oriented computer programming, the wavefunction is an "object," and observable quantities such as the position x and momentum p of the matter represented are applied as *operators* on the wavefunction ψ which then return the desired quantity.

In order to describe the quantum version of a "particle," the wavefunction $\psi(x)$ we seek is one that is localized in space and oscillatory; something qualitatively similar (though in general not identical) to the Gabor wavelet shown in the left frame of Figure 1c if we were to take the horizontal axis as x instead of t – in which case the Fourier transform will give the *wave number* $k = 2\pi/\lambda$, which is the "spatial analogue" of frequency. Such a solution $\psi(x)$ is typically referred to as a "wave packet." As we can see from the right pane, this wave packet comprises a *range* of momentum values, because, given Equation (2.1), the momentum is proportional to k, namely $p = hk/2\pi = \hbar k$, where quantum physicists use the notation $\hbar$ (called "h bar") to denote $\hbar = h / 2\pi$.

Given this range of momentum present in a wave packet, we can gain a qualitative notion of the "uncertainty" relations developed formally below. This range of momentum values at present implies that, for time-dependent cases $\psi(x,t)$, the wave packet will "spread out" as the "fast" parts of the packet outpace the "slow" parts. In other words, the uncertainty in momentum implies an uncertainty in position as the wavefunction develops. The mathematics is somewhat deeper and more subtle, but this qualitative picture can serve in some cases.

2.3. Quantum Uncertainty

The Heisenberg Uncertainty Principle is essentially a statement about Fourier transforms. It is a natural consequence of de Broglie's discovery that matter has wave-like properties. The wavefunction operators that provide position x and momentum p are Fourier transforms of one another, and thus localization in one variable implies a spreading out in another. Formally this is

$$\Delta x \Delta p \geq \frac{\hbar}{2}.$$

Position x and momentum p together form just one example of what are commonly known as pairs of *conjugate variables* arising in quantum physics. Another pair is energy E and time t, which have a similar relation

$$\Delta E \Delta t \geq \frac{\hbar}{2}.$$

If we recall that $E=hf$, we see that this is related to the fact that, in a Discrete Fourier Transform, the frequency resolution Δf is no better than the inverse of the sample length Δt,

$$\Delta f \Delta t \geq 1,$$

which recalls the central observation expressed in Equation (1.1).

A similar uncertainty relation can arise in digital signal processing, when insufficient sampling yields "aliasing" errors [7], and can mean that the uncertainty in measuring a given "macroscopic" signal quantity Q, such as the slope of the signal, is limited by the sample length,

$$\Delta Q \Delta t \geq \varepsilon,$$

again for some constant ϵ determined by the specifics of the measurement. Note that this mathematical similarity is not (obviously) due to any deep analogy between discretely sampled signals and any sort of spatial or temporal discretization in the quantum world. Some such theories exist, notably due to luminaries like Carl von Weizsäcker (ur-theory), John Wheeler (pregeometry), David Finkelstein (spacetime code), David Bohm (topochronology) and Roger Penrose (spin networks) [9], however the discretization scale is typically of order of the *Planck length* $L_P = 1.62\text{x}10^{-35}$ m, much shorter than the typical wavelengths of electrons and similar particles.

3. ACKNOWLEDGEMENTS

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