PHY4410 HW 4 Fall 2019
Due Friday Oct 11 by 4:30pm

Homework:

1. Show that for a Gaussian, the average value (e.g. of s) occurs at the same "location" as the maximum value.

2. Suppose $g(N,U) = CU^{3N/2}$ where C = some constant.
   a) Show that $U = \frac{3}{2}N\tau$
   b) Show that $\left(\frac{\partial^2\sigma}{\partial U^2}\right)_N < 0$. This form of g applies to an ideal gas.

3. In Python, write a program which will plot a histogram of sums of rolls of N six-sided dice, such that you demonstrate an exact ensemble, i.e. each possible set of dice rolls is counted once and only once.
   Print out histograms for $N = 3, 6, 10$.

4. Show that, for large N,
$$\sigma(s) \simeq \ln g(N,0) - 2s^2/N \quad \text{(Yes, it's that easy)}$$

5. Prove the "log version" of the Stirling Approximation:
   Using the fact that for integer N, $N! = \int_0^\infty x^N e^{-x} dx$, $\left(= \Gamma(N+1)\right)$
   show that, for large N, $\ln N! \simeq N \ln N - N$.
   Hint: $\log(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots$

Better hint on 5: Much easier to use $\ln N! = \sum_{k=1}^{N} \ln k$, and let the sum become an integral!!