Analytical Mechanics Homework 1: Due Wed Sept 4 by 5pm.

- 1. Using dimensional analysis, derive an expression for the frequency of oscillation of mass *m* oscillating on a spring with spring constant *k*. (Spring constant has dimensions of force/distance.)
- 2. Using dimensional analysis, derive a proportionality for the period *T* of a planet's orbit, using Newton's constant *G*, the mass *M* of the sun, and the length *a* of the semimajor axis of the orbit.
- 3. Using dimensional analysis, develop expressions for the Planck Length and Planck Mass, in a manner similar to the Planck Time we did in class. Evaluate these expressions numerically in MKS units.
- 4. Consider a unit cube with one corner at the origin and three adjacent sides lying along the three axes of a rectangular coordinate system.
 - a. Find the vectors describing the diagonals of the cube, i.e., the diagonals of the *three faces* which intersect the x, y, and z axes.
 - b. What is the angle between any pair of diagonals?"
- 5. A vector in coordinate basis vs. new basis:
 - a. Express the vector $\vec{A} = 3\hat{t} + 4\hat{f}$ in terms of basis vectors $\hat{u} = \frac{1}{\sqrt{2}}(\hat{t} + \hat{f})$ and $\hat{v} = \frac{1}{\sqrt{2}}(\hat{t} - \hat{f})$.
 - b. Draw \vec{A} graphically and show the orientation of both sets of bases relative to \vec{A} .
- 6. For the expression $(R + r)^{-1/2}$ and given $r \ll R$, use Taylor expansion and linearization to find an expression that is linear in r.
- 7. Prove that, for velocity vector **v** and acceleration vector **a**,

$$v \cdot a = v \dot{v},$$

where $v = |\mathbf{v}|$. (Note, \dot{v} is *not* the same as $a = |\mathbf{a}|$. Rather, \dot{v} is the magnitude of the acceleration along its instantaneous direction of motion.) This implies that, for a moving particle, \mathbf{v} and \mathbf{a} are perpendicular to each other if the speed v is constant. (*Hint:* Differentiate both sides of the equation $\mathbf{v} \cdot \mathbf{v} = v^2$ with respect to *t*...and you're basically done.)

8. Prove that, for the vectors **r**, **v**, and **a** (position, velocity and acceleration),

$$\frac{d}{dt}[\mathbf{r}\cdot(\mathbf{v}\times\mathbf{a})]=\mathbf{r}\cdot(\mathbf{v}\times\dot{\mathbf{a}}).$$

9. TODO: Something about angles and cosines and dot products stuff...