Analytical Mechanics Homework 1: Due Monday Sept 12 by 4pm.

1. Using dimensional analysis, derive a proportionality for the period T of a planet's orbit, using Newton's constant G, the mass M of the sun, and the length a of the semimajor axis of the orbit.
2. Using dimensional analysis, develop expressions for the Planck Length and Planck Mass, in a manner similar to the Planck Time we did in class. Evaluate these expressions numerically in MKS units.
3. Problem 1-3 in the Thornton & Marion textbook. Note: You may assume that this transformation simply "swaps" x->y, y->z, z->x. So your matrix will involve only 1's and 0's
4. Problem 1-7 in the textbook. (Note: "diagonals of the cube" means the diagonals of the three faces which intersect the x, y, and z axes.)
5. Prove the "BAC-CAB" rule for the vector triple product…

   $$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathrm{C}(\mathbf{A}\cdot\mathbf{B})$$

   …by using components in a Cartesian basis. *Warning: This will involve lots of opportunities for writing wrong indices; be careful.*
6. Express the vector $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ in terms of basis vectors $\hat{u} = \frac{1}{\sqrt{2}}(\hat{\imath} + \hat{\jmath})$ and $\hat{v} = \frac{1}{\sqrt{2}}(\hat{\imath} - \hat{\jmath})$. Draw $\vec{A}$ graphically and show the orientation of both sets of bases relative to $\vec{A}$.
7. Three vectors, **A**, **B**, and **C** represent three concurrent edges of a parallelepiped. Show that the volume of the parallelepiped is equal to $|\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})|$.
8. Problem 1-19 in the textbook.
9. Prove that, for velocity vector **v** and acceleration vector **a**,

   $$\mathbf{v} \cdot \mathbf{a} = v\dot{v},$$

   where $v = |\mathbf{v}|$. Hence, for a moving particle **v** and **a** are perpendicular to each other if the speed v is constant. (*Hint:* Differentiate both sides of the equation $\mathbf{v}\cdot\mathbf{v} = v^2$ with respect to t. Note, $\dot{v}$ is not the same as $|\mathbf{a}|$. It is the magnitude of the acceleration along its instantaneous direction of motion.)
10. Prove that, for the vectors **r**, **v**, and **a** (position, velocity and acceleration),

    $$\frac{d}{dt}[\mathbf{r} \cdot (\mathbf{v}\times\mathbf{a})] = \mathbf{r} \cdot (\mathbf{v}\times\dot{\mathbf{a}}).$$