

PHY2010 Activity/Homework 5: Logarithms, SIL & SPL

The response of the human ear to sound intensity is roughly logarithmic. Thus we use a log-base-ten scale, called the *decibel* (dB) scale, in which we describe the *ratio* of a given sound intensity I to some *reference* sound intensity I_0 ($I_0 = 10^{-12} \text{ W/m}^2$).

Logarithms:

The operation " \log_{10} ", or just "log" for short, yields "that power of ten which would equal the number inside the log", e.g., $\log 100 = 2$, because $10^2 = 100$. (Note that this implies you can only take the log of a positive number.) Here are some more properties of logs:

- *Inverse*: "log" and "ten to the" are inverses of each other, and undo each other, i.e. for some number x ,
$$10^{\log x} = x \quad \text{and} \quad \log 10^x = x.$$
- *Log of an Exponent*: The exponent of something inside a log operation can be "brought out" as a multiplicative factor, i.e. $\log x^y = y \log x$.
- *Multiplication & Division*: The log of a product is the sum of the logs, and the log of a fraction is the difference of the logs:

$$\log(xy) = \log x + \log y \quad \text{and} \quad \log\left(\frac{x}{y}\right) = \log x - \log y.$$

SIL Scale:

The absolute SIL of a sound is given relative to the reference level by the following formula:

$$SIL = SIL_0 + 10 \log\left(\frac{I}{I_0}\right).$$

For any two sounds, their SILs are related by a similar formula:

$$SIL_2 = SIL_1 + 10 \log\left(\frac{I_2}{I_1}\right),$$

which we may also express in terms of their *difference*, ΔSIL :

$$\Delta SIL = SIL_2 - SIL_1 = 10 \log\left(\frac{I_2}{I_1}\right).$$

SPL Scale:

A related scale is the Sound Pressure Level (SPL) scale, in which the pressure fluctuation Δp of a sound is compared to a reference pressure fluctuation $\Delta p_0 = 20 \mu\text{Pa} = 20 \times 10^{-6} \text{ Pa}$.

$$SPL = SPL_0 + 20 \log\left(\frac{\Delta p}{\Delta p_0}\right),$$

and the "20" is there because sound energy varies as the *square* of the pressure fluctuation.

Problems:

1. The amplitude response of the human auditory system is also frequency-dependent. Refer to the figure in the textbook showing equal "loudness" curves. In what units is loudness measured? What is the SIL in dB of a 200Hz tone that *sounds as loud* as a 5kHz tone which is playing at 20dB? **Answer: 40-42 dB**
2. (B&S P6.2) A clarinet plays a soft note with an SIL of 40 dB. By what factor must the intensity be increased in order to play the same note at an SIL of 70 dB? **Answer: 1000**
3. A student engineer at the Rock Showcase said that the SIL at front-of-house was 118 dB.
 - a. What sound intensity does that correspond to? **Answer: 0.631 W/m²**
 - b. What pressure fluctuation does an SPL of 118 dB correspond to? **Answer: 15.9 Pa**
4. "Dave, you're talking at 120dB --- the threshold of pain --- and you're 10 feet from my ear. Please back up so that your voice is only 80dB when it reaches my ear." To what distance are you asking Dave to back up? (Assume a "free sound field": no reflections, air absorption or other complications. i.e. use the Inverse Square Law.) **Answer: 1000 ft**
5. Fill in the blank: A difference in SIL of 60dB corresponds to a factor of one million in intensity.