

**PHY2010:  
Physics for Audio  
Engineering Technology**

**Laboratory Manual**

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*Thanks to Profs. Magruder, Cotrill, Tough  
and all my PHY2010 students, past & present,  
for assistance in developing the content of PHY2010.*

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## Welcome and Orientation

Welcome to the Physics for AET Lab. In this lab, we'll be performing experiments dealing with the foundations of acoustics: harmonic motion, waves, sound speed and intensity, to name a few. The following are some of the guiding principles we will use to direct our investigations.

### Lab as a "Simplified Space":

Real-life audio environments (e.g., a live sound venue) tend to be rather complicated spaces, with multiple physical processes occurring simultaneously. In this lab, we will investigate *simpler* systems individually, for which we are primarily interested in one process at a time. This type of *reductionism* is typical in physics, where we often seek to understand physical phenomena in terms of simple "building blocks".

### Theory vs. Observation:

Usually we will have some theoretical model of how the system behaves, such as a mathematical formula relating the speed of waves on a string with the tension in the string and the mass per unit length of the string. Such models constitute *idealizations* which may not in fact take all the relevant factors into account (e.g., whether or not the string "stretches" while it's being used).

Thus, while we would like agreement between our theoretically-motivated *hypotheses* and our observations, disagreements are bound to occur. In such cases, we will not view the theoretically-predicted outcomes as "correct" and our data as "incorrect," rather they are simply "different." We will then seek to identify any *reasons* by which the disagreement may occur.

### Accuracy & Precision:

In order to speak of how well any given observations "agree" with a hypothesis or with other measurements -- i.e. how *accurate* the measurements or model may be --- we need to have some idea of how *precise* the measurements are. Perhaps there's enough "slop" in the measurements (e.g. due to a very poor instrument) that a *range* of values could actually be produced by the same experiment. In fact, the best way to get a handle on this is to *repeat* the experiment multiple times, i.e. to make multiple measurements of the "same thing," and calculate the *average* or *mean* of the various data values, as well as the *standard deviation*. The latter will give us an idea of the *precision* of the measurement, which we can then use to express an expected *range* of acceptable data values.

### "Error":

The causes of disagreement between a hypothesis and observation can be manifold. In general, we will try to identify, as specifically as possible, *what* these causes are and *how* they may have affected the results. For example, "The reason the sound intensity didn't fall off as the reciprocal of the distance squared is that the reflections off the walls were filling the room, resulting in a less-than inverse-square fall-off."

*One form of error which we will avoid referring to is so-called "human error,"* because this term is vague and begs many questions: What kind of error did you make? How did this affect the results? If you know you made an error, why didn't you repeat the experiment correctly?

With these concepts in mind, we're ready to begin!

## **Rules of the Physics Lab**

Be aware that the rules of the lab are for your safety and the safety of others. These rules are based on the guidelines set forth by the EPA and OSHA. Failure to abide by these rules violates Belmont, EPA and OSHA policies.

1. Always use appropriate safety equipment as designated by the lab instructor.
2. Always wear appropriate clothing, including closed-toe shoes. NO sandals, flip flops, etc are permitted. "No shoes, (no shirt,) no service."
3. Long hair must be restrained.
4. Book packs, bags, purses, coats, etc. must not obstruct walkways.
5. ABSOLUTELY NO food or drink in the lab. This includes food, water bottles or drinks in book bags, purses, coats, etc.
6. Follow all the instructions of the instructor and/or assistant.
7. Report any accidents or spills to the instructor.
8. You may only work in the lab during your assigned lab time. Make-up labs may be arranged at the discretion of the instructor in the event of an excused absence.
9. Your work area and common areas must be clean before you leave the lab.
10. All waste must be discarded in appropriate container.
11. Know the locations of all exits, fire extinguishers, fire blankets, eyewashes, and safety showers.

## Notice of Safety and Consent

### *Important Safety notes:*

While every reasonable precaution has been taken to ensure your safety during this lab, when working with physics laboratory apparatus it may still be possible to invent ways to injure yourself or others in the lab. The following guidelines will help to make for a safe and enjoyable lab experience:

- Follow the posted "Rules of the Physics Lab" at all times.
- Don't run with the scissors. In fact, no running in the lab.
- The 500g and 1kg can leave a nasty dent in the floor or your foot if they fall, so make sure they're secured.
- If we're dealing with suspended metal wires (e.g. in the Sonometer Lab), don't put so much pressure on the wire that it might snap and poke your partner's eye out.
- In PHY2010, common sense should keep you safe.
- Note that if you break a "fancy" piece of equipment such as a microphone or SPL meter, you may be required to pay to have it replaced.

"I have read and understood the safety concerns associated with this lab, and will abide by these and the other laboratory procedures outlined by the instructor."

---

**Signed**

---

**Date**

---

**Printed Name**

**Reference: Using Excel**

We will frequently rely on Microsoft Excel to speed up our data analysis. You may initially feel more comfortable doing every single calculation yourself using a calculator. As you become accustomed to using Excel for data analysis, you will find it offers a huge amount of labor-saving functionality, which will in later labs prove essential to finishing the lab in a timely manner.

Here are a few tips for the common functions we'll be using:

**Entering Data:**

Place your data in columns, using a row above the data to *label* the columns, and include the units in the top row. In the data columns themselves, put *only* the numbers, with no letters or units.

Example:

Distance (cm)	Voltage (mV)
1	87.4
2.5	55.6
4.2	23.5

**Performing Calculations:**

In any cell, you can type the symbol "=" to indicate that what follows is to be a *formula*. Then you can refer to other data cells by referencing those cells' column-letter and row-number. For example, to divide the value in cell B2 by the value in cell A2 and place the answer in cell C2, simply select cell C2 and enter "=B2/A2" and press return.

*Copying formulas:* This is where the real labor-saving comes in. Using the above example, if you want the same calculation to be performed for rows 3 through 20 (i.e., taking the value in column B and dividing it by the value in column A), simply select the cell where the formula has been entered already (in this case cell C2), press Control-C for "copy", then highlight cells C3 through C20, and press Control-V for "paste". You'll find that the same formula has been entered in all those cells, except that Excel was smart enough to match the row numbers in the formula with each row you selected!

**Plotting Graphs:**

Arrange the columns of data such that the values to be placed along the vertical axis are immediately to the *right* of the values to be placed on the horizontal axis (the "versus" data). Then simply use the mouse to draw a box around all the data values, press the "Graph" button. For almost all our graphs in this class, you'll want to make an "XY Scatter" plot, which is about the 4th or 5th type of graph on the list.

*Don't forget to label your x and y axes, and include the units as well!*

**Fitting Lines:**

Often we'll fit a line to our graph. Excel refers to these as "Trendlines." To add a trendline, right-click on one of the data points in the graph, and select "Add Trendline" in the popup menu. You'll likely want to report the slope and/or y-intercept of the line, so select "Show Equation on Chart." (This may be on the "Options" sub-menu.) Press "Ok", and you'll have a line!



## Lab: Data Analysis with Excel

**Purpose:** To gain basic skills with data analysis via the labor-saving utility Microsoft Excel.

**Equipment:** Laptop with Excel, printer

### Introduction:

Often in this class, we will take a set of measurements and perform repeated calculations with the data. Rather than entering every single set of values into our calculators and doing the calculations manually for each set, we can use a spreadsheet program like Excel to save time and labor.

Furthermore, the tools of graphing and line-fitting often provide us with insight into the physics we're observing, so being able to use these tools is an essential skill for this course.

The data we'll be using today will be simulated data for next weeks' lab, namely measurements of Hooke's Law,  $F=-kx$ .

### Procedure:

Refer to page 7 of the Laboratory Manual as needed to complete the tasks below.

1. The displacement of a spring from equilibrium was measured as a function of the mass hung from the spring. For the masses of 20g, 30g, 50g, 70g, 100g and 150g, the displacements were 1.6cm, 2.3cm, 4.1cm, 5.6cm, 7.9cm and 12cm, respectively. Enter these data into columns with the appropriate labels. Also include the data point 0g, 0cm at the top.
2. Using formulas, convert the grams to kilograms and the centimeters to meters, placing these converted data into new columns with new labels.
3. Also convert the mass values (in kg) to forces (in N) by multiplying by the gravitational acceleration  $g= 9.8\text{m/s}^2$ .
4. Make a graph of Force vs. Displacement with the appropriate axes labeled (including units!), and place this graph below your data columns on the same page of the spreadsheet.
5. Fit a line to the data and display the equation on the chart. (The slope of this line tells you what? What are the units of the slope?)

### Conclusion:

Print out your spreadsheet, write your names on it, and also write down on the page the value of the spring constant  $k$  for this simulated spring. Don't forget to include the units!

## Lab: Hooke's Law & SHM

### Purposes:

- To observe the relation between applied force and displacement for a linear restoring force.
- To observe aspects of simple harmonic motion.

### Equipment:

- 1 PASCO® Hooke's Law apparatus
  - "crane" assembly
  - plastic indicator disk
- 1 or more springs
- 1 Set of masses to hang from spring(s)
- 1 Stopwatch
- Laptop with Excel

### Introduction:

In this lab, we will measure the spring constant of a spring via two different means:

1. Measuring force vs. distance for the spring --- Hooke's Law. Recall that Hooke's Law states  $F = -kx$ . By plotting  $F$  (on the vertical axis) vs.  $x$  (on the horizontal axis), we can find  $k$  by looking at the *absolute value* of the slope of the curve.
2. Measuring the frequency of oscillation about equilibrium for a given mass --- Simple Harmonic Motion. Recall that in Simple Harmonic Motion, angular frequency  $\omega$  is related to the spring constant  $k$  and oscillating mass  $m$  by  $\omega = \sqrt{k/m}$ , and period  $T = 2\pi/\omega$ , thus  $T = 2\pi\sqrt{m/k}$ . Solving this equation for  $k$ ,

$$k = m \left( \frac{2\pi}{T} \right)^2.$$

### Procedure:

1. Attach the indicator disk to one end of the spring, by threading the end of the spring through the "little tiny ring" on one side of the disk. Save the larger "clip" on the other side of the disk for hanging masses.
2. Hang the other end of the spring from the end of the "crane" assembly.
3. Adjust the vertical scale on the crane assembly so the plastic indicator disk crosses the zero line on the scale. This marks the unstretched position of the spring.
4. Measure the spring constant via Hooke's Law:
  - a. Hang 6 different masses from the underside of the indicator disk, and note the change in length of the spring (i.e. the displacement of the disk) for each mass. *Note that unlike the previous lab, you should **not** include the point "0,0" on the graph, as the spring will have a non-linear response at very small displacements.*
  - b. For each mass, calculate the *weight* force that was applied to the spring.  $F = mg$ , where  $F$  is force in Newtons,  $m$  = mass in kilograms, and  $g = 9.8\text{m/s}^2$ .
  - c. In Excel, make a plot of force  $F$  versus displacement  $x$  for each mass.
  - d. Right-click on one of the data points, and select "Add Trendline..." Fit a line to these data, and obtain the slope of this curve: You will need to go into "Options" in order to have the slope displayed; select "Show equation on chart". The slope you

obtain another measurement of the spring constant. What units does this number have?

5. Measure the spring constant via Simple Harmonic Motion:
  - a. Select some mass or combination of masses such that the spring hangs down "quite a bit" (e.g. 4 to 8cm lower than the unstretched position): You want to have enough space "below" so the masses don't hit the table at the bottom of the oscillation, and enough space "above" so the coils of the spring don't hit each other at the top of the oscillation. You may want to turn the "crane" so it hangs over the edge of the table, and use the 1kg mass to brace the base of the crane so it doesn't fly off the table.
  - b. *Note that the neon disc assembly adds approximately 11.5g to the system. This does not affect the results of Experiment 1 (why?), but for Experiment 2 it can. Thus you should either remove the disc assembly and go with the "bare mass" or else be sure to add 11.5g when considering the "m" of the system.*
  - c. Raise the mass-spring system to some maximum height (but not so high that the spring's coils touch) and get ready release it.
  - d. Using the stopwatch time the oscillation for fifteen cycles. Divide this time by the number of cycles you observed to calculate the period of oscillation. Do this two more times and *average* the period you get for all 3 trials.
  - e. Using this average period and the mass you used, calculate the spring constant. This is your final value of the spring constant.
  - f. Make a second measurement of just the *period* of oscillation, using a larger or smaller *amplitude* than you did before.
  - g. Also vary the mass used and measure the period of oscillation.
6. Answer the remaining questions!

**Hooke's Law / SHO Data Sheet**

Name \_\_\_\_\_

Names of Partner(s): \_\_\_\_\_

**Spring used:** Manufacturer's value of spring constant k: \_\_\_\_\_ (units?)

**Measurement 1: Force vs. Displacement** (Use  $g = 9.8\text{m/s}^2$ )

(You may enter your data directly into table in an Excel spreadsheet.)

Don't include (0,0) this time

Mass m	Displacement x	Force (weight) $F = mg$

From line fit of F vs. x graph:  $k =$  \_\_\_\_\_

**Measurement 2: Period of Simple Harmonic Motion**

Note: You should either add 11.5g to mass you're hanging, or remove the neon disc assembly.

Mass used	Period (for 1 cycle)	Average Period $T_{avg}$
	Trial 1:	
	Trial 2:	
	Trial 3:	

Use  $T_{avg}$  to find k.  $k =$  \_\_\_\_\_

Same thing, amplitude is now larger/smaller (circle one):    Larger    Smaller

Mass used	Period (for 1 cycle)	Average Period $T_{avg}$
Same mass	Trial 1:	
	Trial 2:	
	Trial 3:	

Use  $T_{avg}$  to find k.  $k =$  \_\_\_\_\_

Finally, try a different mass:

Mass used	Period (for 1 cycle)	Average Period $T_{avg}$
	Trial 1:	
	Trial 2:	
	Trial 3:	



**Questions:**

1. *How* do your various measurements for  $k$  agree (or disagree)? How do they agree (or disagree) with the number for  $k$  given by the manufacturer?
2. Any guesses as to why or why not?
3. With the mass in simple harmonic motion, how did the period depend on the amplitude?
4. How did the period of oscillation depend on the mass?

**Lab: Damped, Driven SHM**

Name(s): \_\_\_\_\_

**Purposes:**

- To investigate weakly damped oscillating systems under the influence of driving forces
- To demonstrate the phenomena of resonance

**Equipment:**

- 4 signal generators
- 3 oscilloscopes
- 2 AC Voltmeters
- Resonance Tube & microphone
- Two vibrating LC circuits
- Hanging mass on spring, with magnet, and coil driver
- Optional: Stopwatch

**Introduction:**

A weakly damped oscillator oscillates at its natural frequency  $\omega_0$  and its amplitude decays with a coefficient  $\gamma$ , i.e.

$$x(t) = Ae^{-\gamma t} \cos(\omega_0 t)$$

The oscillator can be driven at some other frequency  $\omega$  and the resulting steady-state amplitude will be a function of  $\omega$ , i.e.  $A(\omega)$ . When  $\omega = \omega_0$ , resonance occurs, at which the amplitude is a maximum.

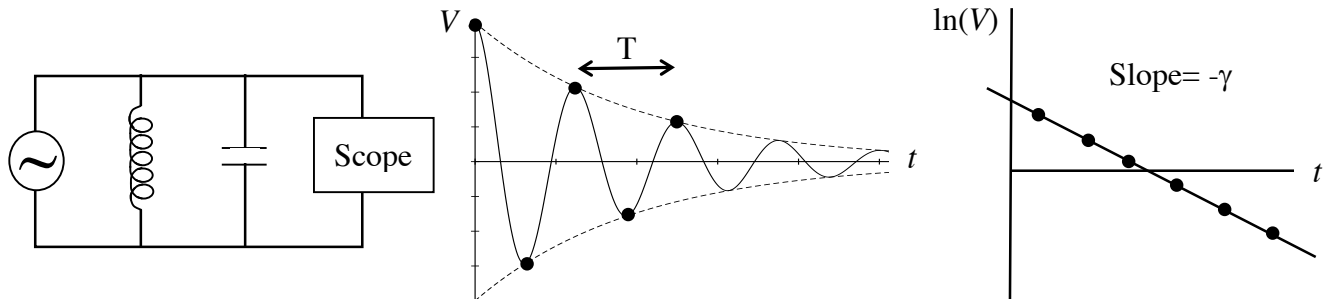
The dimensionless **Q factor** of an oscillator is a measure of how finely-tuned its resonance peak is, and is also a measure of how damped it is.

$$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\Delta \omega} = \frac{\omega_0}{2\gamma}$$

We will use the Q factor to *compare* different oscillating systems.

**Experiment 1: Damping in an Oscillating Circuit (A parallel LC "tank" circuit)**

Electronic circuits made up of an inductor "L" and a capacitor "C" function exactly like simple harmonic oscillators, where the oscillating quantity is the electric charge.



1. Drive the system with a square wave, which will produce a series of damped transient responses. Measure voltages and times using the oscilloscope.

2. Find the period of T oscillation; this will give you the natural frequency  $\omega_0 = 2\pi/T$ .
3. Obtain voltage & time values (i.e. coordinates) for the first 5 extrema (“peaks”/“troughs”)
4. Take the absolute value, and calculate the natural log (“ln”) of the voltage values.
5. Plot the natural log (“ln”) of the voltage vs time.
6. Find the slope of this latter graph; this will give you the damping constant  $\gamma$  (in  $\text{sec}^{-1}$ ).
7. Use this to get the Q value for the oscillator

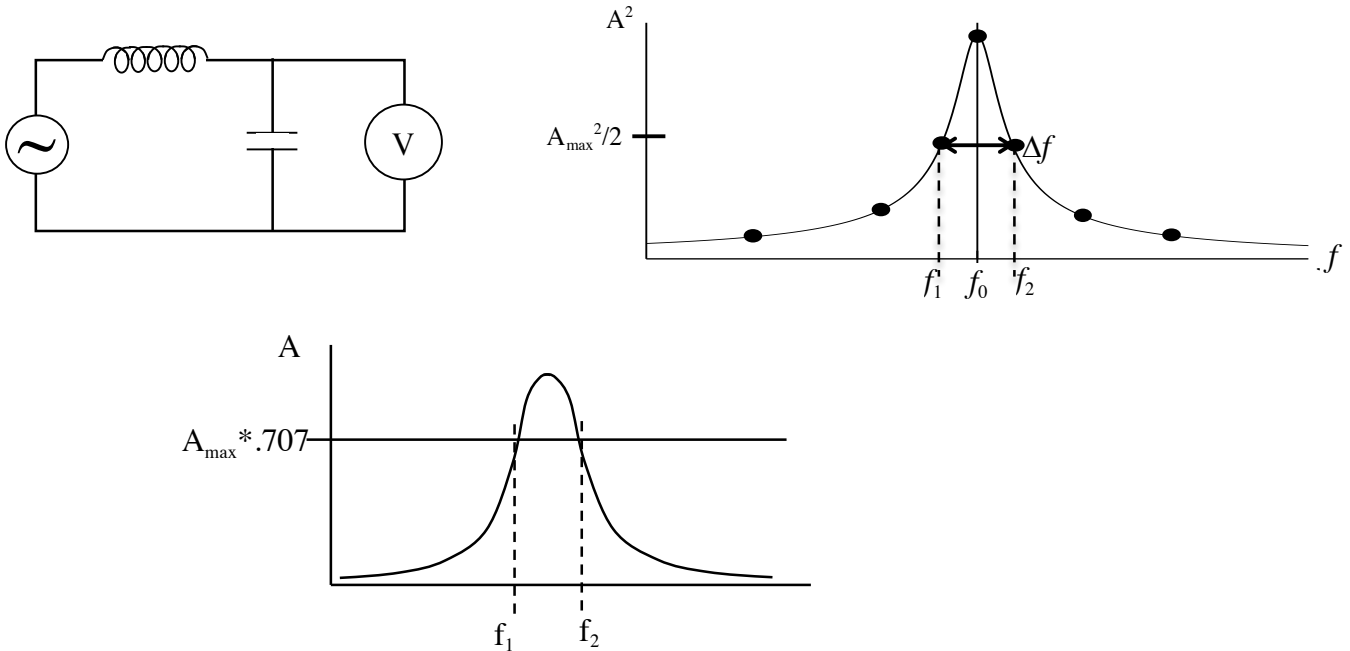
Period T = \_\_\_\_\_

Natural Frequency  $\omega_0 =$  \_\_\_\_\_

Damping constant  $\gamma =$  \_\_\_\_\_

Q factor = \_\_\_\_\_.

**Experiment 2: (2 Stations) Resonance in an Oscillating Circuit (a series LC circuit)**



1. Drive the circuit with sine waves of various frequencies. Look for the frequency which gives you the largest amplitude. This is the natural frequency  $f_0$ .

$f_0 =$  \_\_\_\_\_  $A_{\text{max}} =$  \_\_\_\_\_  $A_{\text{max}} * .707 =$  \_\_\_\_\_

$f_1 =$  \_\_\_\_\_  $f_2 =$  \_\_\_\_\_  $\Delta f = f_1 - f_2 =$  \_\_\_\_\_

Q = \_\_\_\_\_

2. Using Excel, make a plot of  $A^2$  vs.  $f$  for seven values of  $f$ , showing the shape of the resonance peak. Be sure to show the maximum and the two half-max points.

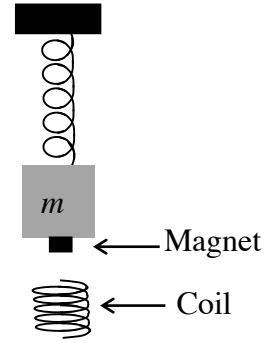


**Experiment 3: Driven Mass on a Spring.**

Try to find  $f_0$  for the mass-spring system. (Use a stopwatch if you like.)

$$f_0 = \underline{\hspace{4cm}}$$

Below 10Hz, there are two *other* types of resonant motion. What are they and what are their resonant frequencies?



**Experiment 4: Resonance Tube**

Find the resonant frequency / frequencies when the plunger is set at 25cm.

Also find the "width at half max"  $\Delta f$ . Use these to obtain Q value(s) for the system.

$$f_0 = \underline{\hspace{2cm}} \quad A_{\max} = \underline{\hspace{2cm}} \quad A_{\max} * .707 = \underline{\hspace{2cm}}$$

$$f_1 = \underline{\hspace{2cm}} \quad f_2 = \underline{\hspace{2cm}} \quad \Delta f = f_1 - f_2 = \underline{\hspace{2cm}}$$

$$Q = \underline{\hspace{2cm}}$$

**Questions:**

1. How did your measurements of  $\omega_0$  agree or disagree between Experiments 1 & 2?
  
2. Which system had the highest Q factor?

## Lab: Speed of Sound

### Purpose:

- To gain experience in measuring the speed of sound in air

### Equipment:

- Laptop computer
- Thermometer
- Experiment 1:
  - Oscilloscope
  - 2 Powered lavalier mics, with 1/8"-to-BNC connectors
  - Signal Generator
  - Speaker
  - Meter Stick
- Experiment 2:
  - Digital Audio Workstation software (e.g., Pro Tools, Audacity, or anything is fine)
  - Audio interface (e.g. Focusrite Scarlett 2i2)
  - 2 microphones (e.g. Audix TR-80 or TM-1)
  - 2 Mic stands
  - Tape Measure

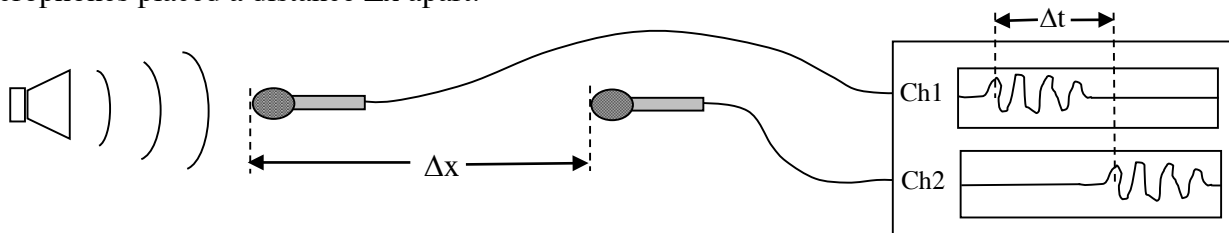
### Introduction:

The speed of sound in air is temperature-dependent, and can be expressed via this formula:

$$v_s = 331 \text{ m/s} + T(0.6 \text{ m/s/}^\circ\text{C}),$$

where  $T$  is the air temperature in degrees Celsius.

We will measure the sound speed by recording the time delay  $\Delta t$  of a signal between two microphones placed a distance  $\Delta x$  apart:



### Procedure:

#### Experiment 1: With the Oscilloscope

1. Place the speaker at one end of the table, and connect the signal generator to it.
2. Lay the meter stick along the table, with one end in front of the speaker.
3. Attach both mics to the oscilloscope, as channels 1 and 2.
4. Turn the mics on
5. Set the oscilloscope Mode to "Dual", so that the traces from channels 1 and 2 are both displayed on the screen.
6. *Important: Make sure the scope's "Var Sweep" knob is turned all the way clockwise.*
7. Make sure you're getting a signal from the mics on the screen, by talking into the mics.
8. Channel 1 mic at the end of the meter stick closest to the speaker.
9. Measure the air temperature

10. Turn the signal generator on, and set it to a square wave between 200 and 500 Hz.
11. Take the Channel 2 mic and, starting from the same location as the other mic, move it back along the meter stick, and at various points record the distance and the time difference between the two signals. (You may find it helpful to pull out the little knob inside the big Volts/div knob for channel 2, which will magnify the signal.)
12. Convert your distance & time measurements to meters and seconds, respectively.
13. Make a graph of distance along the meter stick vs. the time difference in the signals. ("Insert Chart", "XY Scatter", etc.)
14. Add a Trendline to your graph, and select "Display Equation on Chart" under Options.
15. This slope is the speed of sound!

### **Experiment 2: With the Pro Tools Rig**

1. Connect the audio interface to the laptop, insert the iLok key, and start up Pro Tools.
2. Connect the Mics to the audio interface, and give them Phantom Power
3. Place the mic stands some distance apart -- we suggest 4 or 5 meters.
4. Set up a Pro Tools session in which both mics are recording to separate tracks simultaneously. Choose the highest sample rate available. (May be as low as 44.1kHz.)
5. Do a trial recording just to make sure that the signals from the mics are reaching the computer, and set the levels appropriately.
6. Measure the distance between the mics, using the tape measure.
7. Measure the air temperature
8. Start a recording, and make five or six hand claps near one of the mics.
9. Stop the recording, and go back and measure the time difference between the two tracks for each clap. You may want to switch the display from "Time" to "Samples", measure the number of samples between each mic's recording. (If 192kHz is the sample rate, what is the time per sample?)
10. Average these times, and use the distance between the mics to compute your sound speed.

### **Analysis:**

Calculate the *percent error* between your measured values and the expected values, given by

$$\% \text{ error} = \frac{|\text{Measured}-\text{Expected}|}{\text{Expected}} \cdot 100 ,$$

where the vertical bars denote the absolute value.

### **Conclusions:**

Answer the questions at the end.

**Speed of Sound Data Sheet**      **Name(s):** \_\_\_\_\_  
 (You may enter your data directly into table in an Excel spreadsheet.)

**Experiment 1:**

Air Temperature: \_\_\_\_\_. Expected sound speed: \_\_\_\_\_

$\Delta t$	$\Delta x$
0	0

From line fit of Distance vs. Time graph:  $v_s =$  \_\_\_\_\_

**Experiment 2:**

Air Temperature: \_\_\_\_\_. Expected sound speed: \_\_\_\_\_

Distance between mics: \_\_\_\_\_.

<b>Claps!</b>	$\Delta t$	<b>Average <math>\Delta t</math></b>
Clap 1:		
Clap 2:		
Clap 3:		
Clap 4:		
Clap 5:		

Speed of sound (using average): \_\_\_\_\_

**Analysis:**

Experiment 1, percent error =  $|Measured-Expected|/Expected (x100) =$  \_\_\_\_\_.

Experiment 2, percent error =  $|Measured-Expected|/Expected (x100) =$  \_\_\_\_\_.

*Questions on next page!*

**Questions:**

1. Comment on how well your measured values agree with each other, and with the expected values. Use the % error calculations to help your description.

2. Which method do you think was more accurate?

3. What factors may have contributed to the variations between your measured values and the expected values?

4. How might your experience with this lab be relevant to your Audio Engineering work?

## Lab: Standing Waves on Strings

**Purpose:** To investigate the behavior of transverse standing waves

### Equipment:

- Sonometer with wire
- Driver Coil
- Signal Generator
- Set of masses
- Digital scale
- Ruler
- Laptop computer with Excel

### Introduction:

The set of resonant frequencies for a string of mass-per-unit-length  $\mu$ , under tension  $T$ , with length  $L$  fixed at both endpoints are:

$$f_N = \frac{N}{2L} \sqrt{\frac{T}{\mu}}, \quad N = 1, 2, 3, \dots \quad (1)$$

The preceding equation expresses all three of **Mersenne's Laws**, which give the following *proportionalities* for the resonant frequencies of a string:

$$f \propto \frac{1}{L}$$

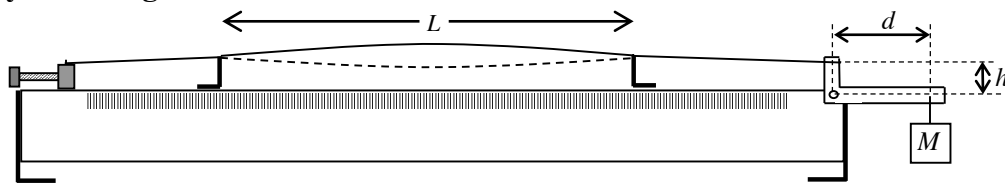
$$f \propto \sqrt{T}$$

and

$$f \propto \frac{1}{\sqrt{\mu}}.$$

We will be using a length of string suspended between two "bridges", like the saddle and nut of the guitar, and attached to an instrument called a *sonometer*. The string is connected on end to a screw and on the other end to a lever.

**Important: Do not push on the lever at the end of the sonometer. You are likely to break your string!**



Sonometer

To drive the string, we will use an electromagnetic driver consisting of a coil wrapped around a small magnet. (This is similar to a guitar pickup, except we will use the coil to *drive* the wire, rather than pick up its vibrations.) The coil will be attached to a signal generator with a tunable frequency and amplitude.

**Procedure:**

1. Remove the string from the sonometer and measure its total length, not including the ball end and eye-hold connector. Also weigh the string on a scale, and subtract 0.9 g for the mass of the ball-end and eye-hole connector. From these, compute the mass per unit length  $\mu$  of the string. Then replace the string on the sonometer.
2. Choose a mass  $M$  (e.g. 200g or 500g) and hang it on the lever. Adjust the sonometer's screw so that the lever arm on the other end is essentially horizontal. Measure the distance  $d$  from the lever's axis to the position of the mass. Also measure the distance  $h$  from the axis to the place where the ball end of the string attaches to the short arm of the lever.
3. Place the wave driver coil directly beneath the string, near one of the bridges.
4. For all of the following measurements, you will find it helpful to **calculate the expected value of the frequency**, according to equation (1), to help you get in the "ball park" of the correct frequency to drive the string. To find the tension in the string, use the equation

$$T = Mg \frac{d}{h},$$

where  $g = 9.8\text{m/s}^2$ ,  $M$  is the hanging mass in kilograms,  $d$  is the distance from the mass to the axis of rotation, and  $h$  is the distance from the string to the axis of rotation.

Calculate the expected frequency  $f_{\text{expected}}$  using the formula at the beginning of the lab.

5. **Frequency vs. 1/Length:** Vary the distance  $L$  between the two bridges which the string goes over, finding the fundamental frequency for each length. *Note: You will need to multiply your measured frequency by 2.\** The distance between the bridges is to be considered as the length  $L$  of the string. *Note: You may find it helpful to "prop up" one side of the bridges on to the "lip" along the sonometer.*
6. Make a plot of frequency verses the *inverse* length, i.e.  $f$  vs.  $1/L$ .
7. **Frequency vs. Harmonic Number:** For a given mass  $M$  and bridge separation  $L$ , measure the frequencies of the first 3 harmonics.
8. **Frequency vs. Sqrt(Tension):** For a given length, vary the masses you place on the lever. Calculate the tension and plot the frequency versus the *square root* of the tension.
9. Fit a line to the graph. The slope of this line is equal to  $1/(2L\sqrt{\mu})$ . Use the slope to calculate the mass per unit length:

$$\mu = \frac{1}{(2L \cdot \text{Slope})^2}$$

**Analysis:**

Calculate the percent error for all your frequency measurements, using the formula

$$\% \text{ error} = \frac{|\text{Measured-Expected}|}{\text{Expected}} \cdot 100$$

You may find it helpful to have Excel do all of these calculations for you. See the sample data sheet, below.

**Conclusions:**

Answer the questions which follow.

\*Because the wire is *non-magnetic*, it is only *attracted* to (and never repelled by) the driver coil, and does so *twice* per cycle of the supplied signal. Thus a "60 Hz" reading on the signal generator corresponds to 120 Hz of string-driving power!

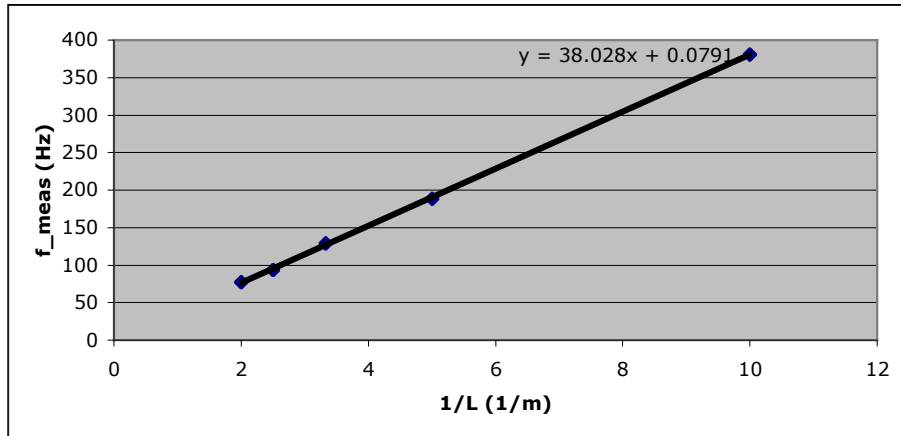


**Sample Data:**

**d (cm) = 5.1      h (cm) = 1.7       $\mu$  (kg/m) = 1.21E-03**

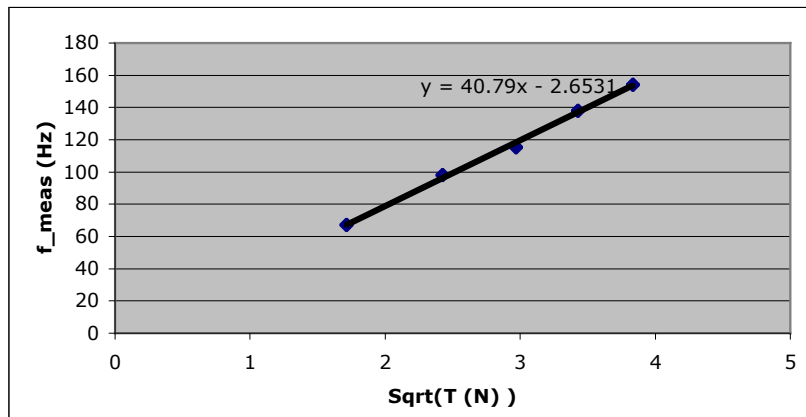
**Frequency vs. 1/Length**

M (kg)	T (N)	L (m)	f_expected (Hz)	1/L	f_meas (Hz)	% err
0.2	5.88	0.1	348.55072	10	380.8	9.252393
0.2	5.88	0.2	174.27536	5	188.5	8.162164
0.2	5.88	0.3	116.18357	3.3333	128.6	10.6869
0.2	5.88	0.4	87.13768	2.5	93.7	7.530979
0.2	5.88	0.5	69.710144	2	77.1	10.60083



**Frequency vs. Sqrt(Tension)**

M (kg)	T (N)	L (m)	f_expected (Hz)	Sqrt(T)	f_meas (Hz)	% err
0.1	2.94	0.4	61.615644	1.7146	67.2	9.063211
0.2	5.88	0.4	87.13768	2.4249	98	12.4657
0.3	8.82	0.4	106.72143	2.9698	115.5	8.22569
0.4	11.76	0.4	123.23129	3.4293	138	11.98455
0.5	14.7	0.4	137.77677	3.8341	154.3	11.99276



**Slope = 40.79 Hz/N**

**† from slope = 0.000939 kg/m**

**Standing Waves on Strings Data Sheet**

Name: \_\_\_\_\_

Name of Partner(s): \_\_\_\_\_

**Apparatus Constants:**

$d =$  \_\_\_\_\_  $h =$  \_\_\_\_\_

Mass of string  $m =$  \_\_\_\_\_ Total length of string  $L_T =$  \_\_\_\_\_

$\mu = m/L_T =$  \_\_\_\_\_.

**Frequency vs. 1/Length:**

Hanging mass  $M =$  \_\_\_\_\_

Length $L$	Frequency $f$

**Frequency vs. Harmonic Number:**

Hanging mass  $M =$  \_\_\_\_\_

$L =$  \_\_\_\_\_

Frequencies of the first 4 harmonics: N=1 Freq = \_\_\_\_\_

N=2 Freq = \_\_\_\_\_

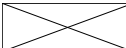
N=3 Freq = \_\_\_\_\_

N=4 Freq = \_\_\_\_\_

**Frequency vs. Sqrt(Tension):**

$L =$  \_\_\_\_\_

Hanging mass $M$	Frequency $f$

From graph: Slope = \_\_\_\_\_ = 

$\mu = 1/(2L*Slope)^2 =$  \_\_\_\_\_

**Questions:**

1. How did the expected frequency values agree or disagree with those you measured?
2. How did the frequencies of higher harmonics agree with the expected multiples of your fundamental frequency?
3. How did your calculated mass per unit length agree with what you measured?
4. *How* would you say Mersenne's Laws have been confirmed or not? Be specific.

## Lab: Standing Waves in Pipes

**Purpose:** To investigate the behavior of longitudinal standing waves

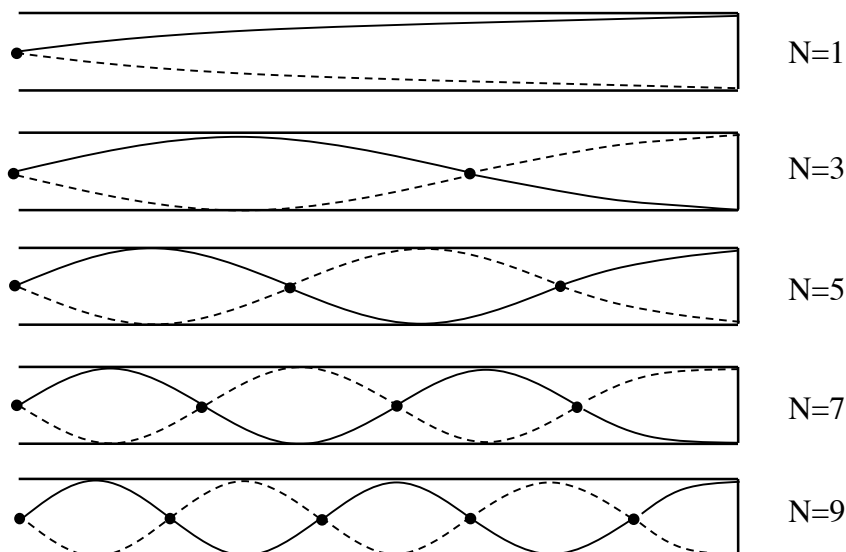
### Equipment:

- Pasco Resonance Tube, with plunger and brass rod
- Laptop computer with Excel
- Signal Generator, and wires with banana ends
- Lavalier microphone, with 1/8" output
- Oscilloscope, with 1/8"-to-BNC adaptor
- Thermometer
- Rubber band or tape
- Verner caliper and/or ruler

### Introduction:

We will excite various resonances in a pipe and try to determine which modes these resonances correspond to, using a variety of analytical tools, including measuring the locations of the pressure nodes. We will be using a microphone to measure the amplitude of the wave. Microphones are sensitive to *pressure*, not displacement. Thus a pressure antinode corresponds to a displacement node, and vice versa.

**Pressure Modes:**  
(Nodes as dark circles)



The finite diameter of the tube actually affects the calculations, and effectively alters the length of the tube. The adjusted length  $L_{adj}$  is given in terms of the physical length  $L$  and diameter  $d$  by

$$L_{adj} = L + 0.26d .$$

In addition to this variation, our tubes are likely to demonstrate a set of resonant modes for a "closed tube" (i.e. one end open, one closed) and a set of resonant modes for a tube with both ends open (or both ends closed). Determining which frequencies correspond to which modes of which system will be the primary challenge of this lab.

**Procedure:**

1. Remove the brass rod from the tube.
2. Back the tube off the speaker by about 4cm. This will enable the modes you observe to be "mostly" those of a pipe with one end open and one end closed.
3. Measure the diameter of the tube using the caliper or ruler.
4. Set up the mic: Thread the microphone all the way through the little hole below the speaker, and extend the brass rod through the same hole. Use the rubber band to affix the mic onto the end of the brass rod. Thus you'll be able to place the mic at different places along the tube by extending or retracting the rod.
5. Set the plunger at 50cm. This will be the length  $L$  of your tube. (Note: In the "Sample Data" below,  $L$  was 43.2cm. You are *not* to use this length.)
6. Extend the mic all the way down the tube until it nearly touches the plunger. This will "guarantee" that you'll be measuring a pressure antinode.
7. Use the thermometer to gain a measurement of the expected sound speed  $v_s$ . Recall that

$$v_s = (331 + 0.6T) \text{ m/s} ,$$

where  $T$  is the temperature in degrees Centigrade.

8. Turn on the oscilloscope and the signal generator and try to find the lowest resonant frequency possible for your "closed tube" by varying the frequency and looking for the largest amplitude.
9. Vary the mic position and record the position of the node. If it's "outside" the tube, give it a negative position.
10. Do the same for the next 7 resonant frequencies of the pipe. For each one, begin with the mic against the plunger to find the resonance, then move the mic along the pipe and record the positions of the nodes.
11. Try to identify which modes correspond to those of a closed pipe, by looking at their ratio to the fundamental, and comparing the frequencies for those expected for a closed pipe. The sample data may be of help to you.

**Analysis:**

Your next goal will be to identify which resonances you measured actually correspond to modes of a closed/open tube --- and what  $N$  value is appropriate for the mode. In order to do this, you will find it helpful to (use Excel to) calculate various additional quantities for each measured frequency  $f$ :

- the ratio  $f/f_1$ , where  $f_1$  is the fundamental frequency you measured. This ratio will suggest which "N" this resonance corresponds to, *i.e.* it will help you to "guess  $N$ ."
- the ratio  $f/N$ , which suggests what the fundamental frequency *would* be for the  $N$  you chose
- the frequency  $f_{calc} = Nv_s/(4L_{adj})$  which is the theoretically-predicted value for this mode
- the percent error between the calculated frequency and the one you measured. Minimizing the percent error is another way of "guessing  $N$ ."

In addition, you can obtain a measurement of the sound speed by noting that that distance between two nodes is *half* a wavelength. From this you can get the wavelength  $\lambda$ , and use  $v=f\lambda$  to get  $v_s$ .

You may find that the analysis comes out "cleaner" for high frequencies than for low ones.

**Conclusions:**

Mark with bold type the resonances which you think correspond to modes of a closed tube, and

answer the questions which follow.

**Sample Data & Analysis:**

**Data for tube with one end open, one end closed:**

Cells w/ grey background indicate user input, others are analysis/calculations

Tube length (m) = 0.42                      Temperature (C) = 22  
 Tube diameter (m) = 0.03                      Expected v<sub>s</sub> (m/s) = 344.2  
 Adjusted L (m) = 0.43

f (Hz)	Nodes at (m)	f / f <sub>1</sub>	N=?	f / N	f <sub>calc</sub>	% err	v <sub>s</sub> (m/s)
<b>189</b>	<b>-0.01</b>	<b>1</b>	<b>1</b>	<b>189</b>	<b>199.74</b>	<b>5.3792</b>	<b>327.348</b>
323	-0	1.71	2	161.5	399.49	19.147	275.842
<b>553</b>	<b>-0.01 0.295</b>	<b>2.93</b>	<b>3</b>	<b>184.3</b>	<b>599.23</b>	<b>7.7155</b>	<b>337.33</b>
667	-0.01 0.285	3.53	3	222.3	599.23	11.309	393.53
905	-0.01 0.14 0.32	4.79	5	181	998.72	9.3843	325.8
<b>1028</b>	<b>-0.01 0.15 0.315</b>	<b>5.44</b>	<b>5</b>	<b>205.6</b>	<b>998.72</b>	<b>2.9314</b>	<b>339.24</b>
<b>1415</b>	<b>-0.02 0.105 0.23 0.35</b>	<b>7.49</b>	<b>7</b>	<b>202.1</b>	<b>1398.2</b>	<b>1.2006</b>	<b>346.675</b>
<b>1827</b>	<b>-0.01 0.08 0.18 0.27 0.37</b>	<b>9.67</b>	<b>9</b>	<b>203</b>	<b>1797.7</b>	<b>1.6298</b>	<b>347.13</b>
<b>2206</b>	<b>-0.01 0.065 0.14 0.22 0.3 0.38</b>	<b>11.7</b>	<b>11</b>	<b>200.5</b>	<b>2197.2</b>	<b>0.4009</b>	<b>341.93</b>

**Bold** indicates modes identified by student

Average v<sub>s</sub> = 343.744

**Standing Waves in Pipes Data Sheet**

**Name:** \_\_\_\_\_

**Name of Partner(s):** \_\_\_\_\_

**Apparatus Constants:**

$L =$  \_\_\_ 0.500m \_\_\_       $d =$  \_\_\_\_\_.

Adjusted length  $L_{adj} =$  \_\_\_\_\_.

**Standing Waves:**

Frequency $f$	Nodes at:

**Questions:**

1. How did the measured frequency values agree or disagree with those you calculated?
  
2. How did the frequencies of higher harmonics agree with the expected multiples of your fundamental frequency?
  
3. Did you notice that the data became more "sensible" or "worked out better" for some range of frequencies more than others? What were these? Any ideas as to why this may (or may not) have occurred.

**Pre-Lab: Sound Intensity Level & Inverse Square Law**

*Pre-lab will be handed out separately*



## Lab: Sound Intensity Level and Inverse Square Law

### Purpose:

- To gain experience in operating an SPL meter
- To investigate intensity variation with distance from a source

### Equipment:

- SPL Meter & manual
- Signal Generator, and wires with banana ends
- 5" Speaker
- Two-meter stick
- Computer with Excel

### Introduction:

In this lab we will be gaining hands-on experience operating an SPL (Sound Pressure Level) meter, and using it to study the "fall-off" of intensity as a function of distance from the source.

SPL and SIL (Sound Intensity Level) are compatible standards, both defined using a *reference sound* and the *decibel (dB)* scale, which describes the logarithm of the ratio between any given sound and the reference sound. For SIL, the reference intensity is  $I_0 = 10^{-12} \text{ W/m}^2$ , and for SPL, the reference pressure fluctuation is  $p_0 = 2 \times 10^{-5} \text{ Pa}$ . The two scales are designed to be compatible, so that for a given sound, the SIL and SPL values will be *similar*. There are some subtle differences between SPL and SIL, but we will use them somewhat interchangeably.

Recall that SIL is defined via

$$SIL = 10 \log \left( \frac{I}{I_0} \right)$$

So then intensity is given by

$$I = I_0 * 10^{SIL/10}$$

Typically the intensity is proportional to some inverse power of the distance  $r$  from the source, *i.e.*

$$I \propto \frac{1}{r^a}$$

where the exponent  $a=2$  for a free sound field; this is the so-called Inverse Square Law. Since not all sound fields are perfectly "free," it is possible for the exponent  $a$  to deviate from 2. In this lab we will measure the exponent by plotting  $I$  vs.  $r$  and fitting a *power law* curve to the data.

### Procedure:

1. Place the speaker at one end of a long table, and extend the two-meter stick along the table in front of the speaker, with the "0cm" mark touching the base of the speaker.
2. Turn on the SPL meter, and wait for it to "count down" so that it is properly initialized.
3. Choose settings for the SPL meter (A/C, Fast/Slow, Max Hold, etc...). What are your justifications for using those settings? Write your settings and justifications on the data sheet.
4. Record the level of background noise, *i.e.* the "noise floor". If the noise floor is below 40dB, the SPL meter will read "LO." (This is a good thing; just write down "< 40dB".)
5. Attach the signal generator to the speaker. Turn the signal generator on and set the frequency to some value between 1000-2000Hz.

6. With the SPL meter at 10 cm, adjust the amplitude of the signal until the meter reads 95 dB ---  
*Note, the example below shows 92dB, but you are to use 95dB.*
7. Now back the SPL meter off to distances of 15, 20, 25, 30, 40, 50, 60, 70 and 80 cm, recording the SPL at each of these locations. Note: The reading on the meter is likely to fluctuate over time by a few dB (or several dB) at each location. Try to record what you regard to be the "time- averaged" reading at each distance.

**Analysis:**

1. Using Excel, compute intensity values for all SIL data points you recorded.
2. Make a plot of these intensity values vs. Distance.
3. Add a "Trendline" to the plot, but for the "Type," instead of choosing a "Linear" trend, choose "Power." Under "Options," choose "Show Equation on Chart." This equation will show the power law for your data, with an exponent which is likely to be close to 2.

**Conclusions:**

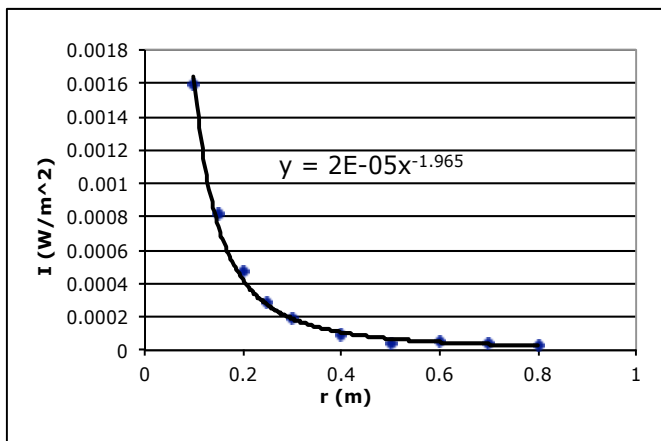
Answer the questions which follow the Data Sheet.

**Sample Data & Analysis:**

Across table, set to auto/fast, f=500Hz

r (cm)	SIL (dB)	r (m)	I (W/m <sup>2</sup> )
10	92	0.1	0.0015849
15	89.1	0.15	0.0008128
20	86.7	0.2	0.0004677
25	84.5	0.25	0.0002818
30	82.7	0.3	0.0001862
40	79.3	0.4	8.511E-05
50	76.8	0.5	4.786E-05
60	77.4	0.6	5.495E-05
70	76	0.7	3.981E-05
80	75.1	0.8	3.236E-05

Inverse power a = 1.965  
 % err in power = 1.75



**SIL & ISL Data Sheet**

**Name(s):** \_\_\_\_\_

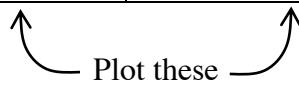
SPL Meter Settings: \_\_\_\_\_

Justification for choosing those settings:

Background noise level = \_\_\_\_\_

Frequency setting (on signal gen.) = \_\_\_\_\_

Distance r (cm)	$\log(r_1 / r)$	SIL_meas'd (dB)	SIL_calc'd (dB)
10 = $r_1$			
20			
30			
40			
50			
60			
80			
100			
120			
150			


  
 Plot these

Inverse power of r:  $a =$  \_\_\_\_\_

% difference between  $a$  and 2 = \_\_\_\_\_

*Questions on next page!*

**Questions:**

1. Why were we "okay" in keeping our distance measurements in centimeters, without converting them to meters? Could we just as easily have measured in inches?
2. How did your measurement compare with that "predicted" by the Inverse Square Law?
3. How many powers of ten was your noise floor below your lowest SPL value?
4. Given your answer to #3, what sort of effect do you think the noise floor had on your measurements?
5. What other systematic or environmental factors may have contributed to the measurements you obtained? And how is the fact that we measured at distances "fairly close" to the speaker relevant?
6. Looking at your measured data, are there any pairs of distances for which you noticed an expected fall-off, e.g. -6dB for doubling the distance or -10dB for tripling the distance? What are these distances?
7. If not already mentioned in your previous answers, how good an approximation is the "free sound field" idea for the region of space in which you conducted your measurements? Why?
8. What would have made this lab more effective, either in terms of obtaining "better" measurements, or in terms of making it more "relevant" for AET?

## Lab: Measuring Reverberation Time

### Purpose:

- To gain experience in measuring reverberation time
- To investigate the effects of absorption on reverberation
- To explore one method of measuring the absorption coefficient of a material

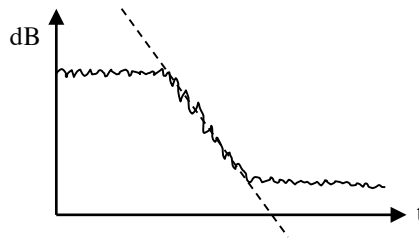
### Equipment:

- Room to measure, e.g. Hitch 101
- Computer with SMAART™ and SMAART Acoustic Tools™ installed
- Audio Interface, e.g. M-Audio FastTrack Pro USB
- Speakers and power amplifier
- Measurement/reference microphone, e.g. Audix TR-80, and XLR cable, and mic stand
- Absorbent materials, e.g. lots of Auralex™ foam squares
- Tape measure and/or yard stick.
- Impulse sound source, e.g. a balloon to pop or a book to slam on a tabletop

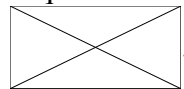
### Introduction:

In this lab, we will measure the reverberation time(s) of a room, with and without acoustical treatment, using both steady state and impulse sound sources. In doing so, we will also obtain a (rough) measurement of the average absorption coefficient of the acoustical treatment material!

The measurement will consist of recording the decay of a sound in the room, and graphically determining the reverberation time. Do so, we graph the waveform, using dB for the vertical scale. Since the sound intensity decays exponentially, the logarithmic dB scale will show a (decreasing) linear trend. Fitting a line to this will yield a measure of the reverberation time:



On the "theory" side, recall that the Sabine Equation can be expressed as



where  $V$  is the volume of the room in  $\text{ft}^3$ ,  $A$  is the area of the surface(s) of the room, and  $a$  is the absorption coefficient of material on those surfaces. For multiple materials, the denominator is the sum of the products  $aA$ , i.e.  $aA = a_1A_1 + a_2A_2 + \dots$

The combined absorption of all materials in the room we can simply call " $X$ ", so that the reverberation time for the untreated room is

$$T_{R1} = \frac{0.050V}{X},$$

whereas for the treated room, it will be

$$T_{R2} = \frac{0.050V}{X + aA},$$

where  $a$  and  $A$  correspond to the coefficient and area of the absorbing material. So, if you *measure*  $T_{R_{Un}}$ , you can solve for  $X$ . Then you can measure  $T_{R_{Tr}}$  and using  $X$  and measuring  $A$  (with a meter stick), you can actually solve for  $a$ , the absorption coefficient of the Auralex!

**Pre-lab:** On the Data Sheet which follows, perform this mathematical operation, i.e., *obtain a formula for  $a$  in terms of  $T_{R1}$ ,  $T_{R2}$ ,  $A$ , and  $V$ .*

### Setup/Preparation:

1. Turn on the computer, let it boot up.
2. Place the speakers near a corner of the room, facing the corner.
3. Connect the power amp, etc, to the audio interface.
4. Plug the audio interface into the computer and turn the power on.
5. On the computer, under "Control Panel," run the control program for the audio interface ("FastTrack Pro") and make sure the driver program shows that the device is "Connected." Do not proceed until it says "Connected." Otherwise, try "ejecting" the USB device (bottom right of the screen, little green arrow icon), and/or rebooting the computer. Good luck.
6. Attach the XLR cable to Channel 1 of the audio interface and to the microphone. Place the mic in the center of the room.
7. On the computer, run SMAART, and select "Pink Noise" near the bottom left of the window. Make sure that sound comes out of the speakers. Adjust the volume so that it's loud, but not so loud that it looks like the speakers are going to burst.
8. Run AcousticTools, and press record. (Choose some filename) Make sure that it actually records a signal from the microphone.
9. You are now ready to make your measurements!

### Procedure:

1. Using a tape measure and a meter stick, obtain an estimate of the volume of the room (in ft<sup>3</sup>).
2. Without any Auralex in the room, Use Acoustic Tools and make one recording containing five decays using a pink noise sound source. Save this recording to a file. Note: To actually *generate* the pink noise, you'll need to run SMAART6 and set the "Generator" to "Pink Noise", and turn it on and off five times while AcousticTools makes the recording. (A bit of a hassle...)
3. Now select "TimeSlice" near the upper left of the window. Choose all the default options that are given. You will be presented with an octave-band filtered signal from your recording.
4. Using the mouse or trackpad, "draw" a line along what you judge to be the linear decay of the sound. Above the picture of the waveform, read off the "T=\_\_\_\_sec" value provided and record it on the Data Sheet.
5. Make measurements for the octave bands 125Hz, 500 Hz, 2000Hz and 8000Hz.
6. Select "Rainbow" and choose 8000Hz under "Band." This shows you simultaneous plots in three adjacent octave bands. What do you notice about  $T_R$  as frequency increases?
7. Also record a decay using the impulse sound source, and save this to a separate file.
8. Perform similar  $T_R$  measurements for the impulsive source recording.
9. Now measure the **total** area of Auralex foam you'll be adding to the room, and record this.
10. Place the Auralex around the room, e.g. on blackboard-sills, tabletops leading against cabinets, etc.
11. Now re-measure the reverb time as in steps 1-6.

**Analysis/Conclusions:** Answer the questions on the Data Sheet.

**Sample Data & Analysis:**  
**Reverb Time**

**Name(s):** \_\_\_\_\_

**Pre-lab:** Solve for  $a$  of Auralex foam in terms of  $T_{R1}$ ,  $T_{R2}$ ,  $A$  and  $V$ :

**Data:**

Room Volume (ft<sup>3</sup>) = \_\_\_\_\_

<b>Without Treatment</b>	$T_{R1}$ 125Hz	$T_{R1}$ 500Hz	$T_{R1}$ 2000Hz	$T_{R1}$ 8000Hz
Pink Noise:				
Impulse:				

Area of Auralex (ft<sup>2</sup>) = \_\_\_\_\_. Flat or Ridged Side Exposed?

<b>With Treatment</b>	$T_{R2}$ 125Hz	$T_{R2}$ 500Hz	$T_{R2}$ 2000Hz	$T_{R2}$ 8000Hz
Pink Noise:				
Impulse:				

**Question 1:** How do your measurements from the different sound sources compare? Which sound source do you think yields a "better" measurement of the room, and why?

**Calculation of  $a$ :** Using your measurements (only from the source you think is "better") and the equation you obtained at the top of the page, calculate the absorption coefficient(s) of the Auralex.

a - 125Hz	a - 500Hz	a - 2000Hz	a - 8000Hz

**Question 2:** If you wanted to obtain more accurate measurements, how would you modify the experimental procedure? (You may consult your text to answer this.)



**Student's Notes:**

